

fs4u

Maths Level 2

Section 1

Working with whole numbers

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A Working with whole numbers

You should already know how to:

- ✓ read, write, order and compare numbers
- ✓ recognise negative numbers.

By the end of this section you will know how to:

- round numbers
- use factors to simplify calculations
- use estimation to check calculations
- use inverse calculations to check answers.
- use the correct level of accuracy for answers.

Place value and rounding

Learn the skill

In the decimal number system all integers are made from the ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.



The value of each digit in a number depends on its position in the number, its **place value**.

Remember

An **integer** is a whole number.

This place-value table shows the number seventeen thousand and sixty-three.

Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
					1	7	0	6	3



You leave a gap or use a comma between groups of three digits.

You write this number as 17 063 or 17,063.

Large numbers are often rounded.



To round a number:

- ▶ Count along to the **last digit** that is needed.
- ▶ If **the next digit** is 5, 6, 7, 8 or 9, round **the last digit** up.
- ▶ If **the next digit** is 0, 1, 2, 3 or 4, leave **the last digit**.

Example 1: 27 687 people watch a rugby match.
Round this number to the nearest thousand.

Write the number in a place-value table:

Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
					2	7	6	8	7

The number in the hundreds column is 6, so round the thousands digit up.

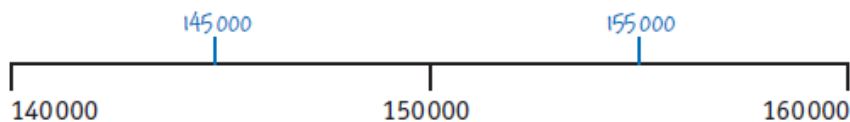
Answer: 28 000

Example 2: A newspaper reported that approximately 150 000 people attended the Glastonbury festival in 2005. What are the possible values of the attendance?

Write the number in a place-value table:

Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
				1	5	0	0	0	0

Assume the number is rounded to the nearest ten thousand. You can show the rounded number on a number line:



Answer: Between 145 000 and 155 000

Remember

A **billion** is one thousand million.
1 billion = 1 000 000 000 or 10

Example 3: The table shows UK consumer spending on credit and debit cards over the festive season in December 2005 and December 2006.

UK consumer spending	2005	2006
Credit and debit cards	£28.5 billion	£31 billion

How much more was spent on credit and debit cards in December 2006 than in

December 2005? The amounts are written in billions of pounds.

You need to work out $£31 - £28.5 = £2.5$ billion i.e. £2 500 000 000

Answer: £2.5 billion

Try the skill

1. A large company announces annual profits of £23 billion. Write the number 23 billion in figures.
2. Our galaxy contains roughly 250 000 000 000 stars. How many billion is this?
3. One of the largest lotto wins was £22 590 829. Round this amount to the nearest £100 000.
4. In 2005–2006 the average attendance at a premiership football match was 34 000 to the nearest 1000. What was the smallest possible average attendance?
5. A population of 3.5 million bacteria increases by 488 000. How many bacteria are there now?

2 Negative numbers

Learn the skill

Negative numbers are used for:

- temperatures below zero
- heights below sea level
- company losses
- overdrawn bank accounts.

▶ To find the **difference** between two numbers:

▶ If the signs are the same, **subtract** the numbers.

▶ If the signs are different, **add** the numbers.

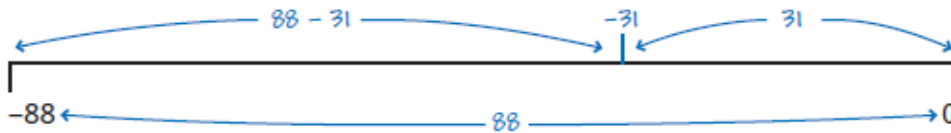
Example 1: A minimum temperature of -88°C was recorded in Antarctica.

Moscow recorded one of their lowest overnight temperatures of -31°C .

What is the difference between the two temperatures?

The lower of the two temperatures is -88°C .

Sketch a number line.



The signs are the same.

You find the difference by subtracting: $88 - 31 = 57$.

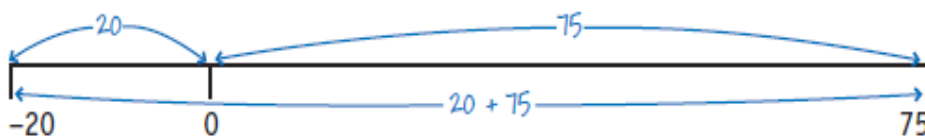
Answer: 57 degrees

Example 2: A girl has an overdraft of £20 on her bank account.

How much must she pay in, so that the balance is £75?

The overdrawn balance is $-\text{£}20$.

Sketch a number line.



The signs are different.

You find the difference by adding: $20 + 75 = 95$.

Answer: £95

Remember

A **loss** or **overdraft** is a **negative** amount of money.

Try the skill

1. On one night in London the temperature fell to 12°C . On the same night in Moscow the temperature fell to -5°C . What was the difference in temperature?

2. A boy started the month with a balance of $-\pounds 25$ in his bank account. He paid $\pounds 120$ out of his account during the month but did not put any money into his account. What was his balance at the end of the month?

3. A company reported a loss of $\pounds 10$ million in 2004. In 2005 it reported a profit of $\pounds 3$ million. What is the difference between the two amounts?

4. The boiling point of krypton is -152°C . The boiling point of radon is -65°C . What is the difference between the two boiling points?

5. In 3 months US energy giant Enron went from a company with assets of $\pounds 62\text{bn}$ to a company with debts of $\pounds 18\text{bn}$. What is the difference between these two amounts?

6. The highest recorded temperature in the UK is 39°C . The lowest recorded temperature in the UK is -27°C . What is the difference between these two temperatures?

7. A bank allows university students a $\pounds 1000$ overdraft facility on their bank accounts. In one month a student was $\pounds 675$ overdrawn. The next month the same student was $\pounds 730$ overdrawn. What is the difference between these two amounts?

Negative numbers on your calculator

The $\boxed{-}$ key on a calculator is used for subtractions. Use the $\boxed{+/-}$ or $\boxed{(-)}$ key on your calculator to calculate with negative numbers.

For example, if you want to enter -5 , press $\boxed{5}$ $\boxed{+/-}$ or $\boxed{(-)}$ $\boxed{5}$

To work out the answer to question 7 on your calculator, key in: $\boxed{(-)}$ \rightarrow 675 \rightarrow $\boxed{-}$ \rightarrow $\boxed{(-)}$ \rightarrow 730 \rightarrow $\boxed{=}$

Try this to check your answer to question 7.

Remember

A **profit** or **credit** is a **positive** amount of money.

Remember

Assets are recorded as positive amounts.

Tip

Only scientific calculators have a $\boxed{+/-}$ or $\boxed{(-)}$ key

3 Factors and multiples

Learn the skill

Multiplications and divisions can be broken down into stages using **factors**.

For example, $3 \times 2 = 6$ so multiplying by 6 is the same as multiplying by 3 and then by 2.

▶ The **factors** of a number are the numbers that will divide into it exactly.

For example, the factors of 12 are: 1, 2, 3, 4, 6, 12.

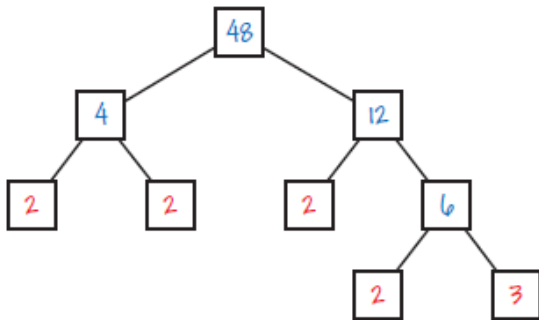
▶ A **prime number** has exactly two factors: itself and 1.

▶ The prime numbers UP TO 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

You can write a number as the product of its **prime factors** using a **factor tree**.

Example: Write 48 as the product of its prime factors.

Draw a factor tree. Split the number into any factor pair and carry on until you have only prime numbers at the end of each of the 'branches'.



Answer: $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

2 is a factor of 48, so 48 is a **multiple** of 2.

▶ A **multiple** of a number can be divided exactly by that number.

For example, 8, 12 and 16 are all multiples of 4.

Here are some quick ways of checking for factors and multiples:

- 2 is a factor of even numbers ending in 0, 2, 4, 6, or 8
- 3 is a factor when the digits add up to 3, 6 or 9
- 5 is a factor when the number ends in 5 or 0
- 9 is a factor when the digits add up to 9
- 10 is a factor when the number ends in a 0.

Tip

1×12 , 2×6 and 3×4 are all the possible *factor pairs* of 12.

Remember

A **product** is the result of a multiplication.

Tip

Check that each branch of your factor tree ends with a prime number.

Tip

$2^4 = 2 \times 2 \times 2 \times 2$

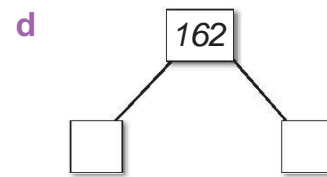
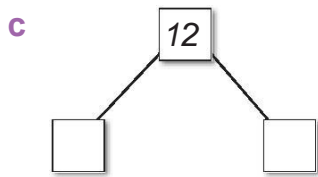
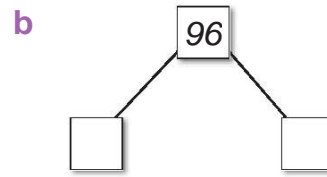
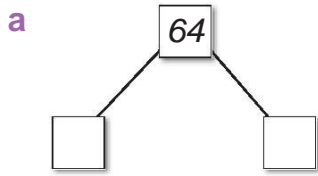
Tip

Keep adding the digits until you get a single digit.



Try the skill

1. Write each of these numbers as a product of its prime factors. The factor trees have been started for you.



2. Write down which of these numbers are multiples of:

A 2 B 3 C 5 D 9

The first one is done for you.

a 78 A and B

b 145 _____

c 261 _____

d 523 _____

e 630 _____

f 936 _____

g 1275 _____

h 3147 _____

i 6795 _____

4 Estimating and checking

Learn the skill

You can improve your accuracy if you check your answers.

There are two main methods: using estimation and using inverse calculations.



You can estimate by rounding to one significant figure (s.f.).

$$126 = 100 \text{ to 1 s.f.} \quad 5.37 = 5 \text{ to 1 s.f.} \quad 87 = 90 \text{ to 1 s.f.}$$

Example 1: A student rents a bedsit for £58 a week. Approximately how much will he pay for the year?

You need to work out 58×52 .

Round each number to 1 s.f.: 60×50 .

$$60 \times 50 = 6 \times 5 \times 10 \times 10 = 30 \times 100 = 3000$$

Answer: £3000

Try to choose numbers with common factors in division problems.

Example 2: A conversion from euros to pounds is €1 = 69 p. Approximately how many euros are equivalent to £140?

You first need to change the pounds to pence by multiplying by 100.

$$140 \times 100 = 14\,000 \text{ pence}$$

Every 69 p is €1 so you need to work out $\frac{14\,000}{69}$.
It is helpful to round 69 to 70.

69 = 70 to 1 s.f. and 7 is a factor of 14.

$$\text{The calculation is } \frac{14\,000}{70} = \frac{1400}{7} = 200$$

Answer: €200

You should check that your answer makes sense. For example, you know that €1 = 69 p so £1 is more than €1 and £140 is more than €140.



You can use an **inverse calculation** to check answers.

Example 3: Twenty-five students each pay 48 pence for a college magazine. What is the total amount spent, in pounds? Use an inverse calculation to check your answer.

The total amount spent in pence = 25×48 . To change pence to pounds, divide by 100.

The calculation is shown in the flowchart.

$$25 \rightarrow \boxed{\times 48\text{p}} \rightarrow \boxed{\div 100} \rightarrow \text{£12} \quad \text{i.e. } \frac{25 \times 48}{100} = 12$$

Check: Start with £12 and do the inverse calculations to arrive back at 25.

$$25 \leftarrow \boxed{\div 48\text{p}} \leftarrow \boxed{\times 100} \leftarrow \text{£12} \quad \text{i.e. } \frac{12 \times 100}{48} = 25$$

Tip

Breaking numbers into factors makes it easier to calculate.



Tip

This could be worked out using decimals. See the section on decimals and money.

Tip

Make sure the quantities are in the same units.

Tip

+ is inverse to -
× is inverse to ÷

Tip

There is more than one way to work out the answer. Can you think of a quicker way?

Try the skill

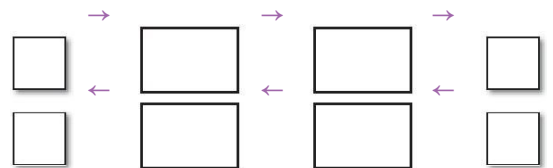
1. The cost of a holiday for 42 students is £9875. By rounding each number to the nearest 10, estimate the cost of the holiday for each student.

2. A man spends £23 per week on travel. Approximately how much does he spend on travel in two years?

3. A dressmaker buys 62 feet of material. 1 foot = 12 inches and 1 metre is approximately 40 inches. Approximately how many metres of material does she buy?

4. Approximately how many miles are equivalent to 162 km? Use the conversion 5 miles are approximately 8 km.

5. Six friends go out for a meal to celebrate a girl's birthday. The cost per meal is £25. Her five friends decide to treat the girl to the meal. Use the first flowchart below to work out how much each of the friends will pay to cover the total cost. Use the second flowchart to show how you would check your result.



Levels of accuracy

Learn the skill

A calculator can be used to check your answers, but when a calculator is used to work out the answers to problems you often need to think about the *level of accuracy* that is needed for the answer. Sometimes this is given in the question.

Example 1: A conversion from kilograms to pounds is 1 kg is approximately 2.2 pounds. How many kilograms are equivalent to 150 pounds to the nearest kilogram?

Every 2.2 pounds is 1 kg so you need to work out $\frac{150}{2.2}$

Using a calculator $150 \rightarrow \div \rightarrow 2.2 \rightarrow =$

gives a calculator display of 68.18181818

Answer: 68 kg to the nearest kg

Sometimes you need to decide on the level of accuracy for yourself, by considering the information you are given and deciding what is appropriate.

Tip

The level of accuracy is sometimes described as the 'degree of accuracy'

Example 2: A conversion from dollars to pounds is \$1 = 47p. How many dollars are equivalent to £300?

First change the pounds to pence by multiplying by 100

$$300 \times 100 = 30\,000\text{p}$$

Every 47p is \$1 so you need to work out $\frac{30\,000}{47}$.

Using a calculator $30\,000 \rightarrow \boxed{\div} \rightarrow 47 \rightarrow \boxed{=}$

gives a calculator display of 638.2978723

The answer can be given correct to the nearest dollar \$638

Or correct to the nearest cent \$638.30

Tip

There is more than one way to work out the answer. Can you think of a quicker way?

Try the skill

Use your calculator to work out the answers to questions 1 and 2 and state the degree of accuracy you have used:

1. Fifteen apples are sold for £2. What is the cost of 1 apple?
2. Three similar onions weigh 1 kilogram. What is the approximate weight of each onion in grams?
3. Fill in the sensible degree of accuracy, in metric units, for the lengths of:
 - a swimming pool – to the nearest _____
 - a book – to the nearest _____
 - an ant – to the nearest _____
 - a train journey – to the nearest _____
4. Eleven friends receive a bill for £170 after a meal at a restaurant. How much should each pay to just cover the bill?
5. A glass holds 400 ml. How many similar glasses can be filled from a jug which holds 3 litres?

Remember

$$1 \text{ kg} = 1000 \text{ g}$$

Remember

The metric units for length are mm, cm, m and km

Remember

Check that the total amount paid is at least £170

Remember

$$1 \text{ litre} = 1000 \text{ ml}$$

5 Tips for calculating

Learn the Skill

Here are some tips to help you do calculations.

1. You can **calculate change** by **counting up** from the cost.

Example 1: A woman's bill in the supermarket is £17.85.
How much change does she receive from a £20 note?

Count up from £17.85 to £20.

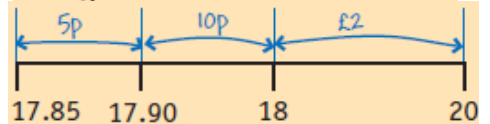
$$£17.85 + 5p = £17.90 \quad £17.90 + 10p = £18 \quad £18 + £2 = £20$$

$$\text{Total change} = 5p + 10p + £2 = £2.15$$

Answer: £2.15

Tip

Imagine the number line:



Tip

This is particularly useful when you are subtracting from a number ending in zeros.

2. You can use the **number line** to help with **subtraction**.

Example 2: Calculate $8000 - 276$.

Subtracting 276 is the same as subtracting 200 then 70 then 6, using the partitioning method.



Answer: 7724

3. You can use **factors** to **multiply** and **divide**.

▶ $4 = 2 \times 2$: to divide by 4, divide by 2 then divide by 2 again.

▶ $5 = 10 \div 2$: to divide by 5, first multiply by 2 then divide by 10.

4. You can use the **lattice** (sometimes called the Chinese method) method for long **multiplication**.

Example 3: What is the total cost of 231 meals that cost £34 each?

You need to work out 231×34 . Set this calculation out as a grid, with the digits of one number at the head of each of the columns and the digits of the other number at the end of the rows as shown here:

▶ Split the boxes inside the grid diagonally as shown and extend the diagonals down outside the grid.

Multiply the number at the top of each column by the number at the end of each row.

Write your answers with two digits in the relevant boxes until the grid is completed.

Starting from the right, add the digits between the diagonal lines remembering to carry any tens to the left.

Write your answers outside the box between the diagonals.

Tip

The answer in this box is the result of multiplying 1 by 3 written with two digits as 03.

2	3	1	×
0	6	0	3
0	8	0	4

7 8 5 4

Tip

The sums are:

$$\begin{aligned} 4 \\ 3 + 0 + 2 = 5 \\ 0 + 9 + 1 + 8 = 18 \\ 0 + 6 + 0 + 1 = 7 \end{aligned}$$

Answer: $231 \times £34 = £7854$

5. You can use **repeated subtraction** for **long division**.

Example 4: Buses that seat 36 people are to be used to transport 1 168 students. How many buses will be needed?

First work out some of the *multiples* of 36.

Subtract multiples of 36 from 1 168.

$$\begin{array}{r} 1168 \\ \underline{720} - \text{20 buses} \\ 448 \\ \underline{360} - \text{10 buses} \\ 88 \\ \underline{72} - \text{2 buses} \\ 16 \end{array}$$

$$\begin{array}{l} 1 \times 36 = 36 \\ 2 \times 36 = 72 \\ 10 \times 36 = 360 \\ 20 \times 36 = 720 \end{array}$$

An **extra bus** is needed for the **16 students** 'left over'.

Total number of buses = $20 + 10 + 2 + 1 = 33$

Answer: 33

Try the skill

Try out each of the tips in these questions.

1. A man's shopping bill is £15.37. How much change will he receive from a £20 note?

2. A shopkeeper buys 4000 England flags. On the first day he sells 437. How many flags are left?

3. a Four friends share a lottery win of £538. How much money do they each receive?

- b Convert 425 miles to kilometres. (Use 5 miles are approximately 8 km.)

4. Three hundred and fifty-two biology students go on a field trip that costs £23 each. What is the total cost?

5. Buses that hold 28 people are to be used to transport 1432 students. How many buses will be needed?

6 Remember what you have learned

First complete this ...

- ▶ An integer is a _____ .
- ▶ The value of each digit in a number depends on its position in the number, its _____ .
- ▶ A _____ is one thousand million: 1 000 000 000 or 10^9 .
- ▶ To round a number:
 - ▶ Count along to the last digit that is needed.
 - ▶ If the next digit is 5, 6, 7, 8 or 9, _____ the last digit up.
 - ▶ If the next digit is 0, 1, 2, 3 or 4, _____ the last digit.
- ▶ To find the difference between two numbers:
 - ▶ If the signs are the same, _____ the numbers.
 - ▶ If the signs are different, _____ the numbers.
- ▶ A loss or overdraft is a _____ amount of money. A profit or credit is a _____ amount of money.
- ▶ The _____ of a number are the numbers that will divide into it exactly.
- ▶ A _____ number has exactly two factors: itself and 1.
- ▶ A product is the result of a _____ .
- ▶ A _____ of a number can be divided exactly by that number.
- ▶ You _____ by rounding to 1 s.f.
- ▶ You can use an inverse calculation to _____ answers.
- ▶ To divide by 4, divide by _____ then divide by _____ again. To divide by 5, first multiply by _____ then divide by _____ .

1. A company makes a profit of £653 172.
What is this figure to the nearest £100 000?
- | | |
|---|----------|
| A | £600 000 |
| B | £650 000 |
| C | £653 000 |
| D | £700 000 |

2. The attendance at a football match was 6500 to the nearest 100.

What is the smallest possible attendance?

- A 6400
- B 6450
- C 6490
- D 6600

3. After a shopkeeper cleans his freezer its temperature is 7°C . The temperature should be -16°C before he puts his frozen food back in.

By how many degrees must the temperature fall?

- A 9°C
- B 11°C
- C 16°C
- D 23°C

4. Some students organise a leavers' ball. The cost of the hall is £200 and the buffet is £15 per person.

If there are 100 people, which of these calculations would you use to find out how much each would pay?

- A $15 \times \frac{200}{100}$
- B $\frac{15 + 200}{100}$
- C $\frac{15}{100 + 200}$
- D $\frac{100}{15 + 200}$

5. A man buys 13 T-shirts that cost £9.86 each.

Which calculation gives the closest estimate to the total cost of the T-shirts?

- A $13 \times \text{£}10$
- B $13 \times \text{£}9.90$
- C $13 \times \text{£}9.80$
- D $13 \times \text{£}9.50$

6. A coach company uses 53-seater coaches to transport passengers between Heathrow and Gatwick airports.

How many coaches would be needed to transport 4862 passengers?

- A 90
- B 91
- C 92
- D 93

7. In one week a shopkeeper sold 451 DVD players for £36 each.

What was the total amount of money the shopkeeper received for the DVD players?

- A £7136
- B £4059
- C £16 236
- D £40 536